LETTERS TO THE EDITOR

To the Editor:

Malhotra and Wasan (Sept., 1987, p. 1533) presented an analysis of the drainage of thin films of foams and emulsions which assumed tangentially immobile surfaces and accounted for the flow in the plateau borders. Their results predict drainage times with a 0.8 power dependence on the film radius. They indicated that all previous theoretical expressions for the drainage of thin plane-parallel liquid films essentially predict a 2.0 power dependence of the thinning line on film radius. If the bubble surfaces are partially mobile, power dependencies of less than two will be obtained.

The drainage time for a viscous plane-parallel film with completely mobile surfaces is given by

$$t = 6\mu/\Delta p \ln (h/ho) \qquad (1)$$

where Δp is the pressure difference between the film and the stagnant fluid, μ is the fluid viscosity, and h is the film thickness. Note that there is no dependency of drainage time on film radius, R^0.

Marrucci (1969) presented an analysis of coalescence with two stages of film drainage: a rapid thinning to a quasi-equilibrium film thickness and a diffusion controlled thinning until rupture. The thinning time for a constant-radius film was given as:

$$t = xRf/2D \qquad (2)$$

where x is a diffusional distance at the film border, R is the film radius, D is the diffusion coefficient, and f is a function of the radius of curvature and the fluid properties. This dependence on film radius, R^1, is close to the 0.8 power dependence of previously reported data.

For a surface to be tangentially immobile, the tangential stress due to viscous forces must equal the stress caused by a gradient of surface tension at every radial position. Diffusion of surface-active molecules will occur to reduce the gradient of surface tension therefore imparting some mobility to the surface. Only when the diffusion of surface-active molecules is small compared to the thinning rates will the surfaces approach the limit of being immobile.

The important point is that if the bubble surfaces are not completely immobile in the tangential direction, the film thinning time will have some dependence on the film radius of less than two. Diffusional-controlled thinning gives close to the correct power dependence of drainage times on film radius. Experimental work is required to discriminate between existing theoretical models.

Literature cited

Malhotra, A. K., and D. T. Wasan, "Effect of Film Size on Drainage of Foam and Emulsion Films," *AIChE J.*, **33**, 1533 (Sept., 1987).

Marrucci, G., "A Theory of Coalescence," *Chem. Eng. Sci.*, **24**, 975 (1969).

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Reply:

Dr. Bousfield makes the observation that if liquid-film surfaces are mobile, then the thinning time will depend on the film radius by a power less than two. In making this observation, he cites the analysis of Marrucci. We would like to point out that the previous analyses of Zapryanov et al. (1983) and Ivanov et al. (1976) also predict a less than two dependence of thinning time on the film radius for a fully mobile film. This condition, however, is rarely observed experimentally and is only valid for pure liquid films which do not contain surfactants.

For systems containing high surfactant concentration, such as those used by Manev et al. (1984), the film surfaces are immobile since the value of surface tension gradient (estimated by Malhotra and Wasan) was found to be greater than 10^6 $\mu Nm^2/$mol. These sytsems were analyzed by Malhotra and Wasan (Sept., 1987a, p. 1533) by accounting for flow both in the film as well as in the Plateau borders. Their analysis predicts the correct dependence of film radius, R, on the film drainage time, $t \sim R^{0.8}$. As stated by these authors, none of the previous analyses takes into account flows in the film as well as in the Plateau borders. For a comprehensive analysis of the effects of surfactants on the drainage of foam and emulsion films, the reader is referred to a recent paper by Malhotra and Wasan (1987b).

Literature cited

Zapryanov, A., A. K. Malhotra, N. Aderangi, and D. T. Wasan, "Emulsion Stability: An Analysis of the Effects of Bulk and Interfacial Properties on Film Mobility and Drainage Rate," *Int. J. Multiphase Flow*, **9**, 105 (1983).

Ivanov, I. B., and T. T. Traykov, "Hydrodynamics of Thin Liquid Emulsion Films from Pure Liquids," *Int: J. Multiphase Flow*, **2**, 397 (1976).

Manev, E. D., S. V. Sazdanova, and D. T. Wasan, "Emulsion and Foam Stability—The Effect of Film Size on Film Drainage," *J. Colloid Interf. Sci.*, **97**, 591 (1984).

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